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Basic Accelerator Related rf Bench Test

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1. Beadpull Technique

Standing wave structure

Slater's theorem [1] states, in cavity, the fractional change in frequency is proportional to the fractional change in stored energy which in turn is proportional to the square of the field times the polarizability of the perturbation

$$\frac{\Delta\omega}{\omega} = -\frac{\varepsilon\alpha_e}{4U}|E|^2 \qquad (1.1)$$

where E is the unperturbed electric field, U is the stored energy in the empty cavity and α_e is electric polarizability of the perturbing object provided that the perturber is dielectric material. In [2] Carter derived formula for perturbation of a pill-box cavity by a dielectric sphere and analyzed and verified its accuracy under some assumptions.

Traveling Wave structure

For traveling wave structure, Steele [3] developed a non-resonant perturbation theory, which determines the phase and field strength at a point inside a microwave structure by measuring the reflection produced at the input port by a perturbing object

$$\Delta S_{11} = -\frac{j\omega}{2P_i}(\varepsilon\alpha_e|E|^2 - \mu\alpha_m|H|^2)e^{j2\varphi} \qquad (1.2)$$

where P_i is input RF power, and α_e and α_m are tensor polarizabilities. For dielectric bead in our perturbation measurement, $\alpha_e=4\pi r^3$ (r is radius of bead), and$\alpha_m=0$ [4]. In [5], the authors suggested the test bench in which Steele's theory was applied. More recently, the method is used to measure traveling wave muffin-tin accelerating structure at SLAC [6].

Bead-pull Experiment by Network Analyzer

By HP8510C network analyzer we made a HP BASIC® code to control step motor and read data from analyzer. Code for standing wave structure can record each resonance frequency reading while bead moving through the cavity. In traveling wave code, output file is S11 reading (in format of real and imaginary number) at each bead moving step. Then, based on formulae shown above we can obtain the accelerating field profile at once.

2. R/Q Measurement

Using an axial field beadpull measurements data we can calculate the ratio R/Q. Take a cavity R/Q measurement as an example as follows. We know

$$\frac{R}{Q}=\frac{\left(\int_L E(z)dz\right)^2}{\omega U} \qquad (2.1).$$

Combining with equation (1.1) we have

$$\frac{R}{Q}=\left(\int_L \sqrt{\frac{4\Delta f(z)}{2\pi f^2 \varepsilon \alpha_e}}dz\right)^2 \qquad (2.2),$$

and for the dielectric bead $\alpha_e = 4\pi r^3$.

3. Q Measurement

Accurate Q measurement of microwave resonators is a big topic (refer to [7]), and all the methods are based on measuring S21 curve of the cavity under test. Here, we just present rough Q measurement by reading S11 curve because that there is only one coupling port available in lot of cases.

In frequency domain, we have

$$|S_{11}|=\sqrt{\left(\frac{\beta-1}{\beta+1}\right)^2+\frac{4\beta}{(1+\beta)^2}\sin^2\varphi} \qquad (3.1),$$

where β is coupling coefficient, and transient angle φ is defined by

$$\tan\varphi = Q_L\left(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right) \qquad (3.2).$$

By setting $\delta=\frac{\omega-\omega_0}{\omega_0}$, the equation (3.1) be replaced as

$$|S_{11}|=\sqrt{\left(\frac{\beta-1}{\beta+1}\right)^2+\frac{4\beta}{(1+\beta)^2}\frac{(2Q\delta)^2}{1+(2Q\delta)^2}} \qquad (3.3).$$

Case 1: Critical coupling---β=1

Then, equation (3.3) is simplified to be

$$|S_{11}|=\sqrt{\frac{(2Q\delta)^2}{1+(2Q\delta)^2}} \qquad (3.4).$$

Set 2Qδ=1, then we have

$$Q=\frac{1}{2\delta}=\frac{\omega_0}{2(\omega-\omega_0)}=\frac{\omega_0}{B} \qquad (3.5),$$

where B is bandwidth. And $S_{11}=\sqrt{0.5}=3dB$. That means we can obtain Q by looking for 3dB bandwidth of resonant S_{11} curve of the cavity under test and applying it to equation (3.5).

Case 2: Non-critical coupling---$\beta \neq 1$

The equation (3.3) cannot be simplified, but we still can set $2Q\delta=1$ and use equation (3.5) to get Q. Note that the bandwidth used in equation (3.5) is not 3dB bandwidth. It needs to be calculated by

$$|S_{11}| = 20\log\left(\sqrt{\left(\frac{\beta-1}{\beta+1}\right)^2 + \frac{2\beta}{(1+\beta)^2}}\right) \quad dB \qquad (3.6).$$

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